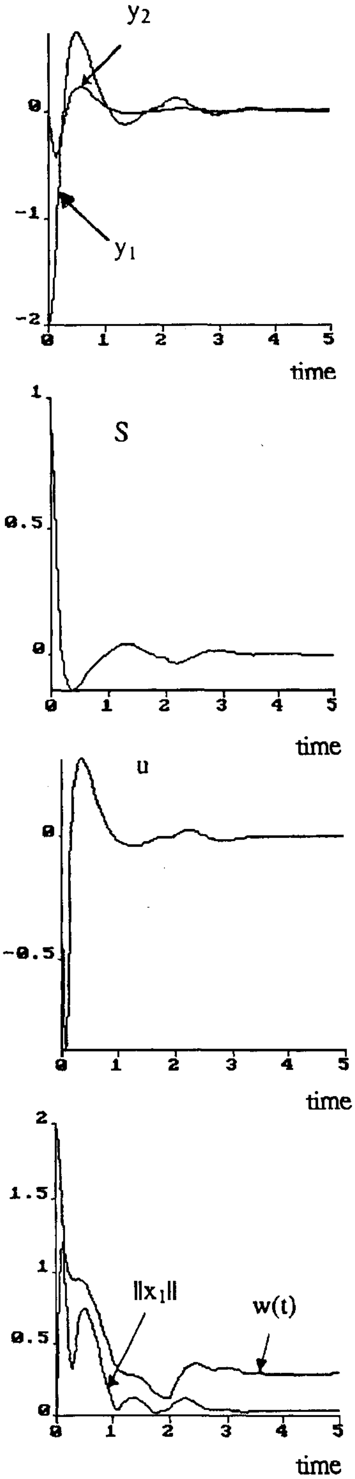

Fig. 2 Performance of our controller.

$-3.06 \pm j3.06$. G is given by $[-0.4635 \quad 1]$. It can be verified that M and N in Eq. (13) satisfy

$$\|M\| < 10, \qquad \|N\| < 2.5$$

The value of K is 0.44. Hence, we arrive at the following controller:

$$u = -(10w(t) + 2.5|S| + 0.44\bar{d})\mathrm{sgn}(s)/6.67 \tag{16}$$

with $\mathrm{sgn}(S) = S/(|S| + 0.5)$ to reduce control chattering. The λ_w in Eq. (10) is chosen to be 3 and the γ in Eq. (10) is set as 1.5. We let $w(0) = 0.01$ since we do not know the initial conditions of y_i. Figure 1 shows the performance of Wang and Fan.[1] Figure 2 shows the performance of our method. It can also be seen that the $\|x_1\|$ is smaller than $w(t)$ after approximately 0.1 s. It is clear that our method is quite good in terms of smaller error and control effort.

Conclusions

We proposed a new sliding control method using output feedback that can guarantee global stability. The current scheme is very simple in structure because no observer is needed. The computational requirement is also very small. It is robust to all initial conditions, mismatched disturbance and parametric uncertainties.

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Parametric Uncertainty Reduction in Robust Missile Autopilot Design

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I. Introduction

THE μ-synthesis design procedure provides a very general framework for the design of robust autopilots.[1–5] The μ-synthesis technique[6] seeks to minimize the structured singular value[7] of the closed-loop transfer function from disturbances and perturbations to reference outputs and the outputs to the perturbations. Optimization is accomplished by alternatively minimizing the infinity norm of the closed-loop transfer function over the set of stabilizing controllers[8] and minimizing the maximum singular value of this transfer function over the set of D-scales.[7] The frequency-dependent D-scales are approximated by rational transfer functions and appended to the plant between iterations.

The μ-synthesis algorithm is not guaranteed to converge to the global minimum, and if it does converge to the global minimum, this minimum is only guaranteed to be optimal in terms of robust performance for two or fewer perturbations.[6,7] Computer-aided design software may, therefore, fail to find a controller that yields robust performance, even when such a controller exists. It is reasonable to expect the μ-synthesis algorithm to perform better if the number of perturbations can be reduced. This hypothesis has

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been found to be correct in a number of missile autopilot design applications.

Missile models tend to have many sources of uncertainty and are especially susceptible to convergence and numerical problems. These problems can be alleviated partially by reducing the number of modeled uncertainties before application of μ-synthesis. This reduction is accomplished by deleting some uncertainties and scaling the remaining uncertainties to compensate for those deleted. After completion of the μ-synthesis design procedure, robust performance is analyzed using the complete set of uncertainties. This approach yields solutions to problems that were previously intractable.

II. Uncertainty Reduction

The number of modeled uncertainties can be reduced by considering the sensitivities of the system eigenvalues to the uncertain parameters. The eigenvalue sensitivity is defined:

$$S_p^{\lambda_k} \equiv \frac{\partial \lambda_k}{\partial p}$$

where λ_k is an eigenvalue of the system, and p is an uncertain parameter in the system. This sensitivity can be computed using the chain rule:

$$S_p^{\lambda_k} = \sum_{j=1}^{n} \sum_{i=1}^{n} \frac{\partial \lambda_k}{\partial a_{ij}} \frac{\partial a_{ij}}{\partial p} \tag{1}$$

where a_{ij} are elements of the state matrix of the system. The first derivative in Eq. (1) can be written

$$\frac{\partial \lambda_k}{\partial a_{ij}} = \left(\psi_k \phi_k^T\right)_{ij} \tag{2}$$

where ψ_k and ϕ_k are the left and right eigenvectors of the state matrix, respectively.[9] The second derivative in Eq. (1) can be approximated numerically as

$$\frac{\partial a_{ij}}{\partial p} \approx \frac{a_{ij}(p + \Delta p) - a_{ij}(p)}{\Delta p} \tag{3}$$

where Δp is a small perturbation of the uncertain parameter. Combining Eqs. (1–3), the eigenvalue sensitivities are approximated as

$$S_p^{\lambda_k} \approx \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\psi_k \phi_k^T\right)_{ij} \frac{a_{ij}(p + \Delta p) - a_{ij}(p)}{\Delta p}$$

The uncertain parameters are organized into groups that affect the same eigenvalues. Uncertain parameters that affect only the system gain, i.e., do not affect any of the eigenvalues, also form a group. Note that some parameters affect multiple eigenvalues and/or the system gain, and are included in multiple groups.

A single parameter from each group is scaled and used in the controller design to accommodate all of the uncertainties in that group. Note that a single parameter is used for a complex conjugate pair of poles. The uncertainty in each group with the largest magnitude of the sensitivity-bound product $|S_p^{\lambda_k} \Delta p|$ is scaled. Additional factors can be considered in this selection: A single uncertain parameter may be used to accommodate all of the uncertainties in two or more groups; the small-perturbation approximation implied by using the sensitivity may be poor for some parameters. The actual shift in pole location with changing parameters may be more useful in selection of a parameter to scale.

The selected parameter is scaled to match the worst-case perturbation of the poles, which is obtained by plotting the poles for all worst-case combinations of perturbations for the group, a total of 2^r combinations, where r is the number of perturbations in the group. Note that the worst-case perturbation of the poles is not guaranteed to correspond to a worst-case perturbation of the parameters, except in the limit as the maximum perturbations go to zero. Additionally, identifying the perturbed pole in high-order systems can be difficult when there are multiple poles in a region. Both of these difficulties are avoided when the perturbations are sufficiently small. Practical experience in a number of missile autopilot designs indicates that although the poles move significantly, they are still identifiable and the pole's worst-case perturbation corresponds to a worst-case perturbation of the parameters. The selected parameter uncertainty is initially scaled as

$$\Delta p_{\text{req}} = \frac{\left|S_p^{\lambda_k} \Delta p\right|_{\max}}{S_p^{\lambda_k}} \tag{4}$$

The parameter uncertainty is then increased or decreased iteratively until the magnitude of the maximum eigenvalue perturbation attributable to the single parameter p equals the maximum magnitude of the worst-case eigenvalue perturbation $|\Delta\lambda_k|_{\max}$. Note that all perturbations are assumed to be complex in the μ-synthesis autopilot design procedure. The locus of eigenvalues that result from the scaled complex perturbation is a circle and its interior (in the limit of small perturbations). All possible perturbations are therefore enclosed within the circle resulting from the single scaled perturbation.

In summary, uncertainty reduction is accomplished in the following steps:

1) Generate the eigenvalue sensitivities and the magnitudes of the sensitivity-bound products for all uncertain parameters in the system.

2) Sort the parameters into groups affecting the same eigenvalues.

3) For each group, select the perturbation with the largest sensitivity-bound product to scale. Trial-and-error selection of the parameter to scale may also prove useful.

4) Compute the worst-case magnitude of the eigenvalue perturbations caused by the combination of all parameters.

5) Scale the maximum perturbation of the selected parameter to yield a worst-case magnitude of the eigenvalue perturbation equal to that found in step 4. This is done iteratively starting with the initial value [Eq. (4)].

A μ-synthesis autopilot design is then generated using only the selected and scaled uncertainties. The complete design is subsequently verified by testing for robust performance against the complete set of uncertainties.

III. Autopilot Example

The uncertainty reduction technique is illustrated by application to a pitch plane autopilot design for the low-drag ramjet missile depicted in Fig. 1a. This autopilot utilizes combustor vectoring as a means of control. The nonminimal-phase airframe, and the significant inertia in the combustor section combine to make autopilot design for this missile particularly difficult. In fact, the direct application of μ-synthesis to this autopilot design did not yield a robust design.

Table 1 Nominal parameters and uncertainties

Parameters	Description	Nominal value	Uncertainty bound
M_α	Coefficient	761	±183
M_δ	Coefficient	−342	±174
Z_α	Coefficient	0.510	±0.173
Z_δ	Coefficient	0.217	±0.089
I_b	Moment of inertia of the main body	201	±41
H_α	Coefficient	11,600	——
H_δ	Coefficient	10,300	——
I_c	Moment of inertia of the cumbustor	14.5	——
B_m	Damping of the servo	1,520	——
L_a	Distance of the accelerometer from the c.g.	1.71	——
V	Missile velocity	1,050	——
g	Gravitational constant	9.81	——

Table 2 Eigenvalue sensitivities

Eigenvalue, λ	Parameter, p	$S_p^\lambda \approx \Delta\lambda/\Delta p$	$\lvert S_p^\lambda \Delta p\rvert \approx \lvert(\Delta\lambda/\Delta p)\Delta p$
$-36.7 \pm 62.7j$	M_α	$-1.6 \times 10^{-4} \mp 3.1 \times 10^{-5} j$	0.0307
	M_δ	$0.0017 \pm 0.006j$	1.04
	Z_α	0	0
	Z_δ	$0.13 \mp 0.05j$	0.0125
	I_b	$-0.0004 \pm 0.0127j$	0.522
27.4	M_α	0.02	3.66
	M_δ	0.003	0.522
	Z_α	−0.5	0.0865
	Z_δ	0.09	0.00801
	I_b	−0.00530	0.217
−27.3	M_α	−0.02	3.66
	M_δ	−0.006	1.044
	Z_α	−0.5	0.0865
	Z_δ	0.17	0.0151
	I_b	0.0128	0.526

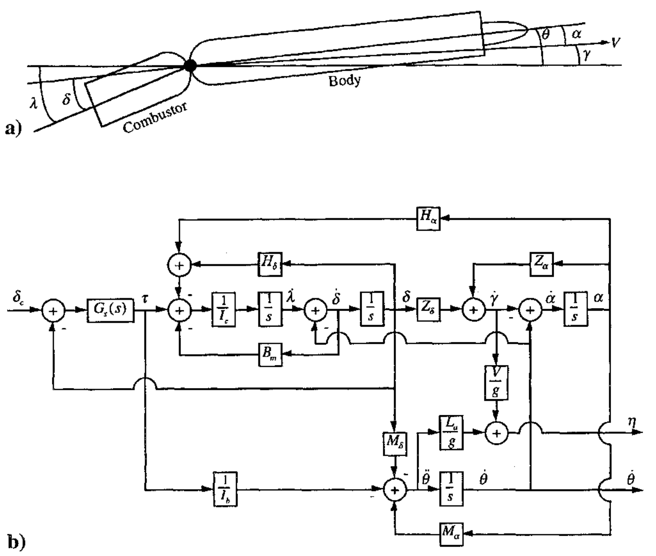

Fig. 1 Combustor-controlled missile: a) configuration and b) block diagram.

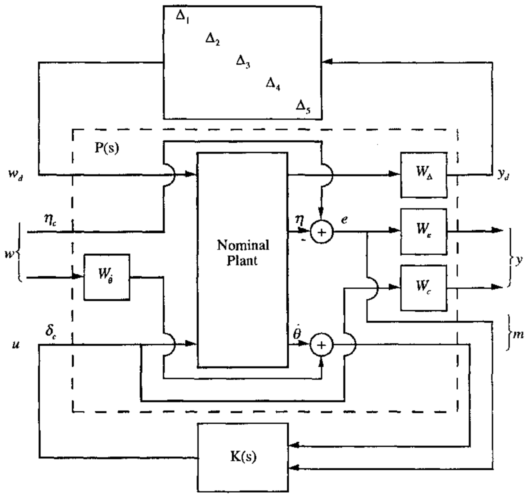

Fig. 2 Performance weighting functions for autopilot design.

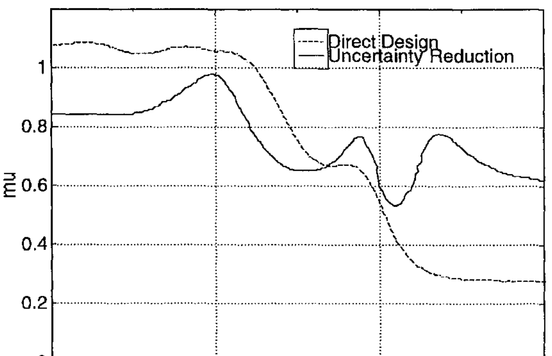

Fig. 3 Structured singular values of the autopilot designs.

A block diagram for this missile, linearized around nominal values of altitude, velocity, angle of attack, and pitch rate, is given in Fig. 1b. The input to the system is a commanded combustor deflection angle δ_c in radians. The measured system outputs are missile acceleration η and pitch rate $\dot{\theta}$. A linear model has been developed for one particular altitude and velocity. Factors affecting the plant parameters are angle of attack and fuel mass, which cause significant changes in five of the parameters Z_α, Z_δ, M_α, M_δ, and I_b. The nominal airframe parameters and associated uncertainties are listed in Table 1, along with a brief description of the parameter. The servo is modeled as a first-order transfer function with a gain of 76,050 and a time constant of 4.44 ms.

The weighting functions that specify the performance of this system are shown in Fig. 2. The weighting function on the error between the commanded acceleration and the actual acceleration is a low-pass filter with a dc gain of 50 and a bandwidth of 0.1 rad/s:

$$W_e(s) = \frac{50(0.025s + 1)}{10s + 1}$$

This weighting function has the effect of requiring that the steady-state error, attributable to a slowly varying acceleration command, be less than 2%. The weight $W_c(s) = 0.1$ is the inverse of the maximum allowable combustor deflection, and guarantees that the deflection command never exceeds this maximum. The weight W_Δ is a diagonal matrix containing the inverses of the bounds on the uncertain parameters, and $W_{\dot{\theta}} = 0.001$ is the bound on the measurement error of the rate gyro (both measurement noise and gyro drift).

An attempt was made to design a robust controller for the plant with all uncertainties using μ-synthesis with D-scales of various orders. The best performance was achieved using D-scales of order two and three. In this case, the μ-synthesis algorithm converged after several iterations, and the structured singular values of the resulting system are shown in Fig. 3. The maximum structured singular value exceeds one, indicating that the closed-loop system does not exhibit robust performance. Uncertainty reduction techniques are, therefore, applied to the model in an effort to obtain a robust controller.

Variations in the uncertain parameters affect four of the five open-loop eigenvalues. The eigenvalue sensitivities for these parameters are listed in Table 2 along with the magnitudes of the sensitivity-bound products. The sensitivities are grouped by eigenvalue in the table. Following the procedures of Sec. II, the maximum eigenvalue change for all possible combinations of parameter variations is

$$|\Delta\lambda_{1,2}|_{\max} = 1.63; \qquad |\Delta\lambda_3|_{\max} = 4.47; \qquad |\Delta\lambda_4|_{\max} = 5.10$$

The parameters for scaling are selected by examining Table 2. The parameter M_δ is selected to scale for the change in λ_1 and λ_2. The parameter M_α has the largest sensitivity-bound product for both λ_3 and λ_4. This indicates that M_α can be scaled to accommodate changes in both of these eigenvalues. The initial scaling values are found using Eq. (4): $\Delta M_\delta = 272$; $\Delta M_\alpha = \text{Max}(213, 243) = 243$. Note that because M_α is being used to accommodate changes in two eigenvalues, the maximum of the two initial scaling values is used. By iterating, the final scaled uncertainty values are $\Delta M_\delta = 273$; $\Delta M_\alpha = 259$.

A robust controller is designed using only the scaled uncertainties in M_α and M_δ. Again μ-synthesis was applied with various D-scale orders. D-scales of order two and three were used in the final design. Figure 3 shows the structured singular values of the system, with this autopilot, subject to the original set of uncertainties. Note that the system meets the specifications for robust performance.

IV. Conclusions

A method of reducing the number of modeled uncertainties is presented. The application of this procedure, prior to this autopilot design using μ-synthesis, yielded improvements in the robust performance of autopilots when compared to the direct application of μ-synthesis. Note that the performance comparison is made with respect to the system subject to the complete set of modeled uncertainties. The uncertainty reduction technique presented was applied to three separate missiles, and yielded reductions of 5–50% in the maximum structured singular value in these cases. Because of space considerations, only one of these autopilot designs is presented above. Although the uncertainty reduction technique presented above has been applied only to autopilot design, it is conjectured that it will prove useful in a wider range of applications.

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Frequency Domain Control of Single Input/Single Output Distributed Parameter Systems

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I. Introduction

ONE of the challenges in structural control is to successfully and confidently apply the control law derived from a discrete, reduced-order model to engineering systems of much higher order. Balas[1] was among the first to show that the spillover associated with discrete system modeling could destabilize a structural control system. There are two types of spillover: observation spillover and control spillover. The former entails the contamination of sensor output through the residual mode dynamics, whereas in the latter, the residual modes are adversely excited by feedback control. Many of the previous studies of the controller design of slewing flexible structures adopted the modal truncation technique in the time domain[2,3] or in the frequency domain.[4,5] Although some systems can be well modeled by a finite number of discretized dominant modes, there are many whose dynamics can only be captured by the distributed parameter model. This Note presents a single input/single output (SISO) controller design for a slewing flexible structure system in distributed parameter model. A stability criterion is developed from the root locus method in the frequency domain, and the criterion is applied to predict the stability of the closed-loop system under a selected control law and a given set of sensor and actuator in non-collocated configurations. It is shown that the control law requires neither modal discretization nor distributed state sensing/estimation; moreover, no functional gain calculation is needed. These advantages are essential for hardware implementation in vibration control application.

II. Stability Analysis

In frequency domain analysis, the open loop transfer function of a SISO system is often written as

$$G_o(\xi, s) = \frac{N_o(\xi, s)}{D_o(s)} \tag{1}$$

where ξ is the spatial coordinate and s the Laplace parameter. Define the transfer function of controller $C(s)$

$$K(s) = k\frac{N_c(s)}{D_c(s)} \tag{2}$$

where $N_c(s)$ and $D_c(s)$ have no common roots, k is a gain parameter, $k > 0$, and the controller is assumed stable. The closed-loop transfer function becomes

$$G_{\text{cl}}(\xi, s) = \frac{N_{\text{cl}}(\xi, s)}{D_{\text{cl}}(\xi, s)} = \frac{D_c(s)N_o(\xi, s)}{D_o(s)D_c(s) - kN_o(\xi, s)N_c(s)} \tag{3}$$

Let C_- denote the open-left-half plane and C_+ the open-right-half plane. The closed-loop eigenvalues will trace the continuous root loci in the complex plane starting from the open-loop poles λ_i and ending at the open-loop zeros z_i as the gain parameter k increases from zero to infinity. The objective is to design a controller such that all closed-loop poles have negative real part thereby staying in C_-. For an undamped flexible structure the root loci will leave the imaginary axis and move into either C_- or C_+ as the control gain

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